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Two-scale simulation of damage in unidirectionally fibre reinforced composite materials

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The finite element analysis of engineering structures usually assumes a homogeneous as well as a continuous medium. The heterogeneity of matter, which is always found on a sufficiently small length scale is neglected by replacing the inhomogeneous medium through a model of a mathematically homogenized material. The macroscopic constitutive behaviour is derived from volume averaging procedures that smear the microscopic heterogeneities.

1 Introduction

The numerical analysis of engineering structures made of fibre reinforced composite materials requires the knowledge of the macroscopic or effective material behaviour. The overall properties can be described in terms of volume averaged quantities that smear the heterogeneities of the microscopic structure and the influence of its defects. The average properties are obtained from a micromechanical analysis of the discontinuous and damaged finescale structure. The efficiently reformulated version of the micromechanically based *Generalized Method of Cells* (GMC) provides the macroscopic tangential constitutive tensor in closed-form. Two-scale simulations become feasible for the analysis of macroscale composite structures considering process depending damage evolution on the microscale of heterogeneous media.

2 Micromechanical Analysis with the Generalized Method of Cells

The Generalised Method of Cells is based on the concept of the representative volume element (RVE) which is a statistically representative patch of the heterogeneous medium. The transition from the micro to the macro level is done by averaging the microfields of stresses and strains over the volume $\mathrm{V_{RVE}}$ of the RVE:

$$\langle \boldsymbol{\sigma} \rangle := \frac{1}{V_{\mathsf{RVE}}} \int_{\mathsf{V_{RVE}}} \boldsymbol{\sigma} \, \mathrm{dV} \quad \text{and} \quad \langle \boldsymbol{\epsilon} \rangle := \frac{1}{V_{\mathsf{RVE}}} \int_{\mathsf{V_{RVE}}} \boldsymbol{\epsilon} \, \mathrm{dV}. \tag{1}$$

In the case of a nonlinear behaviour of the bonding or the nascency and evolution of cracks the average phase strain $\langle \boldsymbol{\epsilon}^{(i)} \rangle$ of the material phase (i) is a nonlinear process dependent tensor functional $\boldsymbol{\mathcal{A}}^{(i)}$ of the macroscopic strain tensor $\langle \boldsymbol{\epsilon} \rangle$:

$$\langle \boldsymbol{\epsilon}^{(i)}(t) \rangle := \boldsymbol{\mathcal{A}}^{(i)} \left[\langle \boldsymbol{\epsilon}(\tau) \rangle \right]_{\tau>0}^{\tau=t} \quad \longrightarrow \quad \langle \dot{\boldsymbol{\epsilon}}^{(i)} \rangle := \frac{d}{dt} \langle \boldsymbol{\epsilon}^{(i)} \rangle = \frac{d}{dt} \boldsymbol{\mathcal{A}}^{(i)} = \frac{\partial \boldsymbol{\mathcal{A}}^{(i)}}{\partial \langle \boldsymbol{\epsilon} \rangle} \frac{\partial \langle \boldsymbol{\epsilon} \rangle}{\partial t} := \tilde{\mathbb{A}}^{(i)} : \langle \dot{\boldsymbol{\epsilon}} \rangle \tag{2}$$

The calculation of its rate in eq. $(2)_2$ leads to the tangential strain concentration tensor $\tilde{\mathbb{A}}^{(i)}$ – see [3] – as defined above. With the help of eq. (2) the rate of the average phase stress tensor $\langle \boldsymbol{\sigma}^{(i)} \rangle$ is computed to:

$$\langle \dot{\boldsymbol{\sigma}}^{(i)}(t) \rangle := \frac{d \langle \boldsymbol{\sigma}^{(i)}(t) \rangle}{dt} = \frac{\partial \langle \boldsymbol{\sigma}^{(i)}(t) \rangle}{\partial \langle \boldsymbol{\epsilon}^{(i)}(t) \rangle} \frac{\partial \langle \boldsymbol{\epsilon}^{(i)} \rangle}{\partial t} = \frac{\partial \langle \boldsymbol{\sigma}^{(i)}(t) \rangle}{\partial \langle \boldsymbol{\epsilon}^{(i)}(t) \rangle} : \tilde{\mathbb{A}}^{(i)} : \langle \dot{\boldsymbol{\epsilon}} \rangle \tag{3}$$

Since N different phases (i) – with volume fractions $c^{(i)} = V^{(i)}/V_{\mathsf{RVE}}$ – are considered, the rate of the average macro stress tensor is calculated from the rate of the macroscopic strain by means of the tangential strain concentration tensors $\tilde{\mathbb{A}}^{(i)}$, the change of the phase stress tensor with respect to the phase strains and the volume fractions:

$$\langle \dot{\boldsymbol{\sigma}} \rangle = \sum_{i=1}^{N} c^{(i)} \, \frac{\partial \langle \boldsymbol{\sigma}^{(i)} \rangle}{\partial \langle \boldsymbol{\epsilon}^{(i)} \rangle} : \tilde{\mathbb{A}}^{(i)} : \langle \dot{\boldsymbol{\epsilon}} \rangle \quad \longrightarrow \quad \tilde{\mathbb{C}}^{*} := \sum_{i=1}^{N} c^{(i)} \, \frac{\partial \langle \boldsymbol{\sigma}^{(i)} \rangle}{\partial \langle \boldsymbol{\epsilon}^{(i)} \rangle} : \tilde{\mathbb{A}}^{(i)} \quad \longrightarrow \quad \langle \dot{\boldsymbol{\sigma}} \rangle = \tilde{\mathbb{C}}^{*} : \langle \dot{\boldsymbol{\epsilon}} \rangle \, . \tag{4}$$

The fibres behave linearly elastic. Since epoxy matrices show significantly nonlinear responses to shear loading, the matrix phase is modelled with a Ramberg-Osgood type of material equation $\epsilon_{ij} = \left[\frac{1+\nu}{E} + \Gamma \left(1 - \delta_{ij}\right) \left(\frac{\sigma_{ij}}{\sigma_0} \right)^{(n-1)} \right] \sigma_{ij} - \frac{\nu}{E} \sigma_{kk}$

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with Young's modulus E, the shear modulus G, Poisson's ratio ν and the shear parameter Γ. The exponent n determines the degree of non-linearity. The GMC approach discretizes the RVE by subdividing the generic cell of the microstructure into rectangular subdomains called subcells. The microscopic displacement field $\boldsymbol{u}(\boldsymbol{x})$ is then piecewisely approximated by linear functions defined on each subcell. The continuity of tractions is ensured along all subcell interfaces. Displacement discontinuities $[\![\boldsymbol{u}(\boldsymbol{x})]\!] = \boldsymbol{u}(\boldsymbol{x}^+) - \boldsymbol{u}(\boldsymbol{x}^-),\ \boldsymbol{x} \in \Gamma = \Gamma_{FM} \cup \Gamma_{MM}$, are conceded to arise at the common boundaries Γ_{FM} of neighbouring fibre and matrix subcells in order to model the imperfect bond of the phases. Predefined subcell interfaces Γ_{MM} of adjacent matrix cells serve also as localisation nuclei for the initiation and growth of crack surfaces within the matrix phase. The traction vector $\boldsymbol{t} = \boldsymbol{\sigma}\boldsymbol{e}_n$ is related to the kinematical counterpart, i.e. the separation vector $[\![\boldsymbol{u}]\!]$, via the traction-separation model given by [4]. This model involves two scalar valued stress and separation measures, both defined over past times $\tau \leq t$:

$$t_v(\tau) = \sqrt{\left(\frac{\langle t_n(\tau)\rangle}{R_\perp}\right)^2 + \left(\frac{t_t(\tau)}{R_\parallel}\right)^2 + \left(\frac{t_b(\tau)}{R_\parallel}\right)^2},\ \|\boldsymbol{u}(\tau)\| = \sqrt{\left(\frac{\langle [\![u_n(\tau)]\!]\rangle}{u_\perp}\right)^2 + \left(\frac{[\![u_t(\tau)]\!]}{u_\parallel}\right)^2 + \left(\frac{[\![u_b(\tau)]\!]}{u_\parallel}\right)^2} \quad (5)$$

Herein the normal and shear strengths are denoted by $R_\perp$ and $R_\parallel$ respectively, whereas the characteristic length parameters $u_\perp$ and $u_\parallel$ determine the ductility of the traction-separation-model. As long as $\max_{\tau \leq t} t_v(\tau) < 1$ holds, no damage of the bond occurs, i.e. the bond flexibility is zero and $[\![\boldsymbol{u}]\!] = \boldsymbol{0}$, otherwise the traction vector $\boldsymbol{t}$ becomes a nonlinear vector-functional $\boldsymbol{f}([\![\boldsymbol{u}(\boldsymbol{x})]\!], q)$ of the current jump vector and the process dependent internal variable $q(t) := \min\{\max \|\boldsymbol{u}(\tau)\|_{\tau=0}^{\tau=t}, 1\}$:

$$\boldsymbol{t} = \boldsymbol{f}([\![\boldsymbol{u}(\boldsymbol{x})]\!], q) = \frac{1 - 3q^2 + 2q^3}{q}\left(\frac{R_\perp}{u_\perp}[\![u_n]\!]\boldsymbol{e}_n + \frac{R_\parallel}{u_\parallel}[\![u_t]\!]\boldsymbol{e}_t + \frac{R_\parallel}{u_\parallel}[\![u_b]\!]\boldsymbol{e}_b\right),\ [\![u_n]\!] > 0 \quad (6)$$

The penetration of phases is numerically suppressed by setting $t_n = K_p[\![u_n]\!]$ with the penalty stiffness K_p. The continuity conditions, imposed on the microfields of stresses and displacements in conjunction with the constitutive equations of the interface model, lead to a system of nonlinear algebraic equations the solution of which finally provides the tangential strain concentration tensors $\tilde{\mathbb{A}}^{(i)}$ for all subcell domains (i), see [5]. Hence, the macroscopic stress tensor $\langle\boldsymbol{\sigma}\rangle$ can be computed by numerical time integration of eq. $(4)_3$ from the effective constitutive tensor $\tilde{\mathbb{C}}$ according to eq. $(4)_2$ for the given macroscopic strain process $\langle\boldsymbol{\epsilon}(t)\rangle$.

3 Numerical Examples

In fig. 1 the results of a longitudinal tension test performed with a thinwalled cylinder, made of a glass-fibre reinforced epoxy material, are presented. The fibres are wrapped round the tube with an angle of $45°$ relative to the loading direction. To this end, the longitudinal loading of the tube causes a torsional deflection of the tube shown in fig. 1b) and 1c). Fig. 1a) compares experimentally observed shear stress vs. shear strain behaviour with the GMC-model predictions using parameters identified by hand. The experimental result is taken from [1].

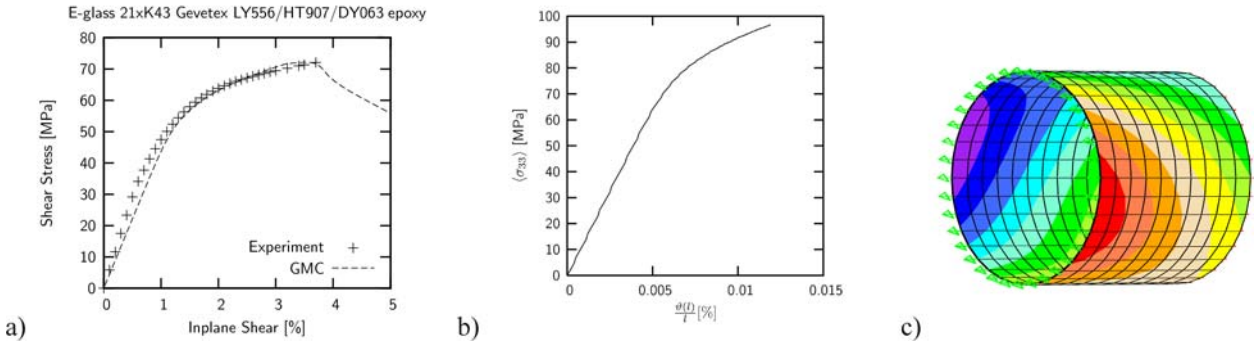

Fig. 1 Parameters of fibres: $c^{(f)} = 0.62$, $K = 80.0$ GPa, $G = 33.333$ GPa. Matrix: $K = 3.35$ GPa, $G = 1.24$ GPa, $n = 10$, $\sigma_0 = 0.08$ GPa, $\Gamma = 2.0$ GPa. Fibre-matrix interface: $R_\perp = R_\parallel = 0.08$ GPa, $u_\perp = u_\parallel = 0.4\mu m$. Matrix cracks: $R_\perp = R \parallel = 0.78$ GPa, $u_\perp = u_\parallel = 0.15\mu m$

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