On Micromechanical Damage Modelling Of Fibre Reinforced Composites

A. Matzenmiller[*1] and **B. Koester**[1]

[1] University of Kassel, Institut of Mechanics, Dep. of Mechanical Engineering, Moenchebergstr. 7 , D-34109 Kassel, Germany

The numerical analysis of engineering structures is usually based upon the assumption of a homogeneous as well as a continuous medium. Macroscopic homogeneity is maintained also for heterogeneous, fibre reinforced composite structures by replacing the inhomogeneous medium through a model of a mathematically homogenized material. The constitutive behaviour is then described in terms of volume averaged quantities that smear the heterogeneities of the microscale.

1 Introduction

The evolution of non-isotropic damage within the different layers of laminated structures is microscopically caused by void nucleations like the debonding of the embedded fibres, the growth of microcracks inside the matrix phase or the breakage of the fibres. Since the development of damage depends on the local loading history, the effective tangential stiffness tensor varies in time for different material points on the macroscale. The average material properties, needed for the macroscale finite element simulation, are obtained by modelling the discontinuous and damaged microstructure with the concept of the representative volume element (RVE). The *Generalized Method of Cells* (GMC), proposed in [1] and [2], is used in order to discretise the representative volume element and to compute the process depending rate form of the constitutive equations as well as the average stress response in closed form for each material point individually – see [5].

2 Micromechanical Analysis with the Generalized Method of Cells

The representative volume element for the micromechanical model of the composite is defined as the spatial subarea of the heterogeneous medium, which is structurally typical for the whole of the composite. The transition from the micro to the macro level is done by averaging the microfields of stresses and strains over the volume $\mathrm{V}_{\mathsf{RVE}}$ of the RVE:

$$\langle \boldsymbol{\sigma} \rangle := \frac{1}{V_{\mathsf{RVE}}} \int_{\mathsf{V}_{\mathsf{RVE}}} \boldsymbol{\sigma} \, \mathrm{dV} \quad \text{and} \quad \langle \boldsymbol{\epsilon} \rangle := \frac{1}{V_{\mathsf{RVE}}} \int_{\mathsf{V}_{\mathsf{RVE}}} \boldsymbol{\epsilon} \, \mathrm{dV}. \tag{1}$$

In the case of a nonlinear behaviour of the bonding or the nascency and evolution of cracks the average phase strain $\langle \boldsymbol{\epsilon}^{(i)} \rangle$ of material phase (i) is a nonlinear process dependent tensor functional $\boldsymbol{\mathcal{A}}^{(i)}$ of the macroscopic strain tensor $\langle \boldsymbol{\epsilon} \rangle$:

$$\langle \boldsymbol{\epsilon}^{(i)}(t) \rangle := \boldsymbol{\mathcal{A}}^{(i)} \left[\langle \boldsymbol{\epsilon}(\tau) \rangle \right]_{\tau>0}^{\tau=t} \quad \longrightarrow \quad \langle \dot{\boldsymbol{\epsilon}}^{(i)} \rangle := \frac{d}{dt} \langle \boldsymbol{\epsilon}^{(i)} \rangle = \frac{d}{dt} \boldsymbol{\mathcal{A}}^{(i)} = \frac{\partial \boldsymbol{\mathcal{A}}^{(i)}}{\partial \langle \boldsymbol{\epsilon} \rangle} \frac{\partial \langle \boldsymbol{\epsilon} \rangle}{\partial t} := \tilde{\mathbb{A}}^{(i)} : \langle \dot{\boldsymbol{\epsilon}} \rangle \tag{2}$$

The calculation of its rate in eq. $(2)_2$ leads to the tangential strain concentration tensor $\tilde{\mathbb{A}}^{(i)}$ – see [3] – as defined above. Since the N different phases (i) – with volume fractions $c^{(i)} = V^{(i)}/V_{\mathsf{RVE}}$ – are still considered here as linear elastic, the rate of the average stress tensor is calculated from the rate of the macroscopic strain by means of the effective constitutive tensor $\tilde{\mathbb{C}}^*$:

$$\langle \dot{\boldsymbol{\sigma}} \rangle = \sum_{i=1}^{N} c^{(i)} \, \mathbb{C}^{(i)} : \tilde{\mathbb{A}}^{(i)} : \langle \dot{\boldsymbol{\epsilon}} \rangle \quad \longrightarrow \quad \tilde{\mathbb{C}}^* := \sum_{i=1}^{N} c^{(i)} \, \mathbb{C}^{(i)} : \tilde{\mathbb{A}}^{(i)} \quad \longrightarrow \quad \langle \dot{\boldsymbol{\sigma}} \rangle = \tilde{\mathbb{C}}^* : \langle \dot{\boldsymbol{\epsilon}} \rangle \, . \tag{3}$$

Only unidirectionally reinforced composites with a periodical microstructure are considered within the framework of the GMC. Due to this restriction the composite structure can be generated by stringing together a sequence of unit cells, consisting of a single fibre, embedded into the surrounding matrix material. The generic unit cell is regarded as a representative volume element (RVE) of the heterogeneous medium. The GMC approach discretizes the RVE by subdividing the unit cell into rectangular subdomains which are referred to as subcells – see fig. 1. The microscopic displacement field $\boldsymbol{u}(\boldsymbol{x})$ is then piecewisely approximated by linear functions defined on each subcell domain. The continuity of tractions is ensured along all subcell interfaces. Displacement discontinuities $[\![\boldsymbol{u}(\boldsymbol{x})]\!] = \boldsymbol{u}(\boldsymbol{x}^+) - \boldsymbol{u}(\boldsymbol{x}^-), \ \boldsymbol{x} \in \Gamma = \Gamma_{FM} \cup \Gamma_{MM}$, are conceded to arise at the common boundaries Γ_{FM} of neighbouring fibre and matrix subcells in order to model the imperfect bond of the phases. Furthermore, predefined subcell interfaces Γ_{MM} of adjacent matrix cells serve also as localisation nuclei for the initiation and

* Corresponding author: e-mail: post-structure@uni-kassel.de, Phone: +00 49 (0)561 804-2044, Fax: +00 49 (0)561 804-2720

growth of crack surfaces within the matrix phase. The traction vector $\boldsymbol{t} = \boldsymbol{\sigma}\boldsymbol{e}_n$ is related to the kinematical counterpart – i.e. the separation vector $[\![\boldsymbol{u}]\!]$ – via the traction-separation model given by [4]. This model involves two scalar valued stress and separation measures, both defined over past times $\tau \leq t$:

$$t_v(\tau) = \sqrt{\left(\frac{\langle t_n(\tau)\rangle}{t_0}\right)^2 + \left(\frac{t_t(\tau)}{\eta t_0}\right)^2 + \left(\frac{t_b(\tau)}{\eta t_0}\right)^2}\ ,\ \|\boldsymbol{u}(\tau)\| = \sqrt{\left(\frac{\langle[\![u_n(\tau)]\!]\rangle}{u_0}\right)^2 + \left(\frac{[\![u_t(\tau)]\!]}{\zeta u_0}\right)^2 + \left(\frac{[\![u_b(\tau)]\!]}{\zeta u_0}\right)^2} \quad (4)$$

Herein the normal and shear strengths are denotet by t_0 and ηt_0 respectively, whereas the characteristic length parameters u_0 and ζu_0 determine the ductility of the traction-separation-model. As long as $\max_{\tau \leq t} t_v(\tau) < 1$ holds, no damage of the bond occures, i.e. the bond flexibility is zero and $[\![\boldsymbol{u}]\!] = \boldsymbol{0}$, otherwise the traction vector $\boldsymbol{t}$ becomes a nonlinear vector-functional $\boldsymbol{f}([\![\boldsymbol{u}(\boldsymbol{x})]\!], q)$ of the current jump vector and the process dependent internal variabel $q(t) := \min\{\max \|\boldsymbol{u}(\tau)\|_{\tau=0}^{\tau=t}, 1\}$:

$$\boldsymbol{t} = \boldsymbol{f}([\![\boldsymbol{u}(\boldsymbol{x})]\!], q) = \frac{t_0}{u_0}\frac{1 - 3q^2 + 2q^3}{q}\left([\![u_n]\!]\boldsymbol{e}_n + \frac{\eta}{\zeta}[\![u_t]\!]\boldsymbol{e}_t + \frac{\eta}{\zeta}[\![u_b]\!]\boldsymbol{e}_b\right)\ ,\ [\![u_n]\!] > 0 \quad (5)$$

If $t_n < 0$, a negative $[\![u_n]\!]$ is numerically suppressed by setting $t_n = K_p[\![u_n]\!]$ with the penalty stiffness K_p. The continuity conditions, imposed on the microfields of stresses and displacements in conjunction with the constitutive equations of the interface model, lead to a system of nonlinear algebraic equations. The solution of the system of equations finally provides the tangential strain concentration tensors $\tilde{\mathbb{A}}^{(i)}$ for all subcell domains (i). Hence, the macroscopic stress tensor $\langle\boldsymbol{\sigma}\rangle$ can be computed by numerical time integration of eq. $(3)_3$ from the effective constitutive tensor $\tilde{\mathbb{C}}$ according to eq. $(3)_2$ for the given macroscopic strain process $\langle\boldsymbol{\epsilon}(t)\rangle$.

3 Numerical Examples

Fig. 1 illustrates the stress response $\langle\sigma_{22}(t)\rangle$ and $\langle\sigma_{21}(t)\rangle$ of the four cells model due to a normal and a shear strain process $\langle\epsilon_{22}(t)\rangle$ and $\langle\epsilon_{21}(t)\rangle$ with all other strain components set to zero in both load cases. The shear modulus of the glass fibres is set to $G_F = 34.0\,GPa$ and the bulk modulus to $K_F = 46.6\,GPa$. The elastic constants of the epoxy matrix are $G_M = 1.3\,GPa$ and $K_M = 3.8\,GPa$. The fibres are $6\mu m$ in diameter and their volume fraction amounts to $c_f = 45\%$. Strength parameters are given in fig. 1.

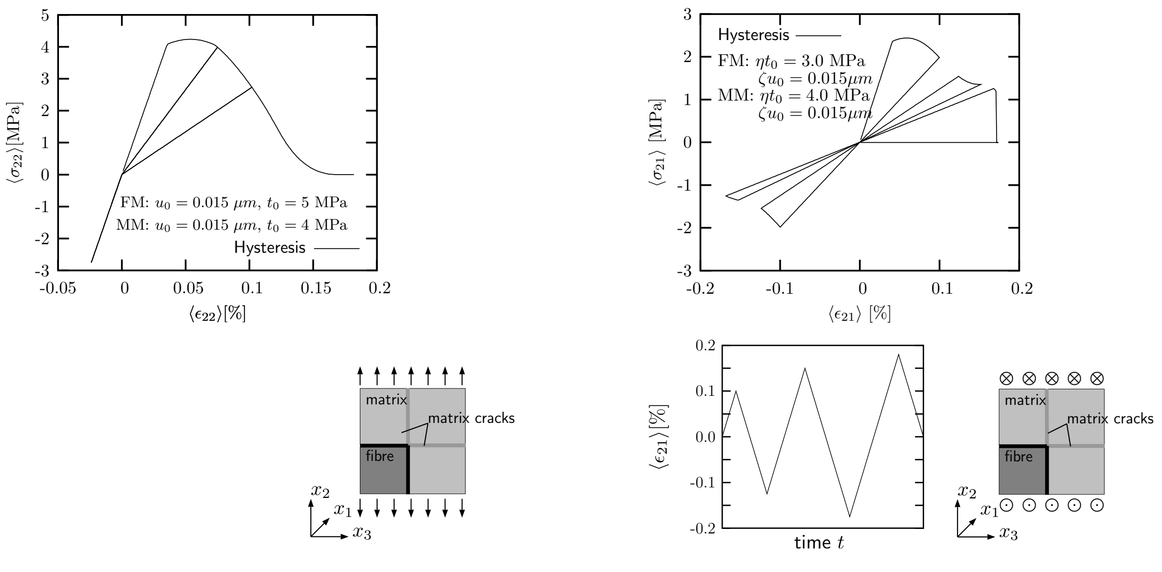

Fig. 1 Stress response $\langle\sigma_{22}(t)\rangle$ and $\langle\sigma_{21}(t)\rangle$ of the unit cell due to a normal and a shear strain process $\langle\epsilon_{22}(t)\rangle$ and $\langle\epsilon_{21}(t)\rangle$.

References

[1] J. Aboudi, Mechanics of Composite Materials - a unified micromechanical approach, Amsterdam: Elsevier, (1991).
[2] M. Paley and J. Aboudi, Micromechanical analysis of composites by the generalized cells method, *Mech. Mat. 14*, p. 127–139, (1992).
[3] R. Hill, Elastic properties of reinforced solids: some theoretical principles, *J. Mech. Phys. Solids 11*, 357-372, (1963).
[4] C.J. Lissenden, An Approximate Representation of Fibre-Matrix Debonding in Nonperiodic Metal Matrix Composites, *Damage and Interfacial Debonding in Composites*, p. 189-212, (1996).
[5] A. Matzenmiller, S. Gerlach and B. Köster, Micromechanically based Finite Element Analyses of Composite Structures. In: M. Schlimmer (Ed.), *Verbundwerkstoffe und Werkstoffverbunde*, p. 461-466, Proceedings of the 15th Symposium *Verbundwerkstoffe und Werkstoffverbunde* of the *Deutsche Gesellschaft für Materialkunde e.V. (DGM)*, April 2005, Kassel.
URL: `http://www.ifm.maschinenbau.uni-kassel.de/ifm/index.htm`